JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER
Vol. 25, No. 3, July–September 2011

Technical Notes

Optimum Shape Design for Three-Layer Interfacial Surfaces

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DOI: 10.2514/1.T3646

Nomenclature

$\mathbf{B}p$	=	control point vector
h	=	heat transfer coefficient
f	=	functional defined by Eq. (8)
k	=	thermal conductivity
M, N	=	basis functions
$T(x, y, z)$	=	temperature
Y_1, Y_2	=	desired domain volumes
Y_3	=	desired system heat flux
λ	=	damping parameter
$\Gamma_1(x, y, z)$, $\Gamma_2(x, y, z)$	=	unknown interfacial surface configuration
ε	=	convergence criterion
$\mathbf{\Psi}$	=	Jacobian matrix
Ω	=	computational domain

Superscript

n	=	iteration index

Subscripts

1	=	region 1
2	=	region 2
3	=	region 3

I. Introduction

INVERSE problems are classified as ill-posed problems, and they are defined as problems when one or more conditions are missing in the corresponding direct problems; for this reason, these kinds of problems are much more difficult to solve than direct ones. When the geometry of the problem is unknown and to be estimated based on the measurement or desired data, it can be called an inverse geometry problem or shape design problem.

Inverse geometry problems, including shape or cavity estimations, have been solved by a variety of numerical methods [1–3]. The authors have used the gradient-based algorithm together with the boundary element technique [4] or commercial code CFD-ACE+ [5] for inverse geometry problems and have published many relevant works on two-dimensional [6–8] and three-dimensional [9,10] applications.

Shape identification problems become shape design problems if the measurement data are replaced by the design or desired conditions. Many applications can be found in the field of engineering: for instance, the design of the shape of a cooling passage in a turbine blade [11], the design of the shape of an interfacial surface between two conductive bodies to minimize the thermal resistance [12], the design of a fuel passage in a proton exchange membrane fuel cell [13], the shape design problem for determining the geometry of the interfacial surface between two conductive bodies in a three-dimensional multiple region domain [14], etc.

Previously [14], the authors used the Levenberg–Marquardt method (LMM) [15], B-spline surface generation [16], the commercial software CFD-ACE+, and the desired system heat flux to determine the shape of an interfacial surface in a two-layer structure and obtained a good estimation. For this reason, the objective of the present inverse geometry problem is that it aims to use the same technologies and then tries to estimate the shapes of interfacial surfaces among three conductive bodies.

The direct problem could be solved by CFD-ACE+, and the calculated results were used in the LMM for shape calculations. Finally, the estimations of this work with different combinations of conductive bodies are illustrated to show the validity of using the LMM in the present three-dimensional shape design problem.

II. Direct Problem

The boundary conditions for regions Ω_1, Ω_2, and Ω_3 are all assumed to be insulated at $x = 0$ and L_1, and $y = 0$ and L_2, and the boundaries at $z = 0$ and L_3 are both subjected to a Robin-type condition with a heat transfer coefficient h and ambient temperatures of $T_{\infty h}$ and $T_{\infty c}$, respectively. The interfacial boundary conditions on $\Gamma_1(x, y, z)$ and $\Gamma_2(x, y, z)$ are both assumed as perfect thermal contact conditions. The mathematical formulation of this problem is given as follows:

$$\frac{\partial^2 T_i(\Omega_i)}{\partial x^2} + \frac{\partial^2 T_i(\Omega_i)}{\partial y^2} + \frac{\partial^2 T_i(\Omega_i)}{\partial z^2} = 0; \quad \text{in } \Omega_i, \quad \text{for } i = 1 \text{ to } 3 \tag{1}$$

subject to the following boundary conditions:

$$\frac{\partial T_i(\Omega_i)}{\partial x} = 0; \quad \text{at } x = 0 \quad \text{and} \quad L_1, \quad \text{for } i = 1 \text{ to } 3 \tag{2}$$

$$\frac{\partial T_i(\Omega_i)}{\partial y} = 0; \quad \text{at } y = 0 \quad \text{and} \quad L_2, \quad \text{for } i = 1 \text{ to } 3 \tag{3}$$

$$-k_1 \frac{\partial T_1(\Omega_1)}{\partial z} = h(T_{\infty h} - T_1); \quad \text{at } z = 0 \tag{4}$$

$$-k_3 \frac{\partial T_3(\Omega_3)}{\partial z} = h(T_3 - T_{\infty c}); \quad \text{at } z = L_3 \tag{5}$$

Interfacial conditions for regions Ω_i and Ω_{i+1} on $\Gamma_i(x, y, z)$ are given as:

$$T_i(\Omega_i) = T_{i+1}(\Omega_{i+1}); \quad \text{on interface } \Gamma_i(x, y, z), \quad \text{for } i = 1 \text{ to } 2 \tag{6}$$

Received 2 November 2010; revision received 12 April 2011; accepted for publication 12 April 2011.

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$$k_i \frac{\partial T_i(\Omega_i)}{\partial n} = k_{i+1} \frac{\partial T_{i+1}(\Omega_{i+1})}{\partial n};$$
$$\text{on interface } \Gamma_i(x, y, z), \quad \text{for } i = 1 \text{ to } 2 \tag{7}$$

here, the subscripts 1, 2, and 3 denote the three different domains with thermal conductivity k_1, k_2, and k_3, respectively, and n denotes the outward normal unit vector. The direct problem can be solved by the commercial package CFD-ACE+, because it has the function of automatic grid generation.

III. Shape Generation for Interfacial Surfaces

The B-spline surface generation technique is considered here to generate the geometry of interfacial surfaces $\Gamma_1(x, y, z)$ and $\Gamma_2(x, y, z)$, because this technique can generalize a smooth surface that satisfies the system requirements and makes a proper shape for an interfacial surface.

Consider the Cartesian product parametric B-spline surfaces given by Γ_p:

$$\Gamma_p(u, w) = \sum_{i=1}^{n+1} \sum_{j=1}^{m+1} B_{p,i,j} N_{p,i,k}(u) M_{p,j,l}(w);$$
$$2 \le k \le n+1; \qquad 2 \le l \le m+1; \qquad p = 1 \text{ or } 2 \tag{8}$$

where

$$N_{p,i,1} = \begin{cases} 1 & \text{if } x_i \le u < x_{i+1} \\ 0 & \text{otherwise} \end{cases}$$
$$N_{p,i,k}(u) = \frac{(u - x_i) N_{p,i,k-1}(u)}{x_{i+k-1} - x_i} + \frac{(x_{i+k} - u) N_{p,i+1,k-1}(u)}{x_{i+k} - x_{i+1}} \tag{9a}$$

and

$$M_{p,j,1} = \begin{cases} 1 & \text{if } y_j \le w < y_{j+1} \\ 0 & \text{otherwise} \end{cases}$$
$$M_{p,j,l}(w) = \frac{(w - y_j) M_{p,j,l-1}(w)}{y_{j+l-1} - y_j} + \frac{(y_{j+l} - w) M_{p,j+1,l-1}(w)}{y_{j+l} - y_{j+1}} \tag{9b}$$

Here, x_i, and y_j, are the elements of a uniform knot vector, k and l are the order of the B-spline surfaces in the u and w directions, and n and m are one less than the number of polygon net points in the u and w directions, respectively. The $B_{p,i,j}$ are the required polygon net points, i.e., the control points. If the control points $B_{p,i,j}$ are given, the interfacial surface data points $\Gamma_1(u, w)$ and $\Gamma_2(u, w)$ can be calculated from Eq. (8). In the present study, the boundary control points of the interfacial surfaces $\Gamma_1(x, y, z)$ and $\Gamma_2(x, y, z)$ are assumed to be fixed, and 16 control points are used to generate each shape of the interfacial surface.

IV. Inverse Design Problem

For the present inverse design problem, the shapes of the interfacial surfaces are regarded as being unknown, and they are dominated by two sets of control points: $\mathbf{B}_1$ and $\mathbf{B}_2$. In addition, the desired domain volumes in Ω_1 and Ω_2, i.e., Y_1 and Y_2, and the desired system heat flux at $z = 0$ (or L_3), Y_3, i.e., $\mathbf{Y} = \{Y_m\}$ and $m = 1$ to I with $I = 3$, are considered available. The solution of this inverse design problem is obtained in such a way that the following functional f is minimized:

$$f[\Gamma_1(\mathbf{B}_1), \Gamma_2(\mathbf{B}_2)] = \sum_{m=1}^{I} [\hat{Y}_m(B_{1,i,j}, B_{2,i,j}) - Y_m]^2 = \Delta^T \Delta;$$
$$i = 1 \text{ to } J, \qquad j = 1 \text{ to } J \tag{10}$$

Here, $2 \times J^2$ represents the total number of control points; $\hat{Y}_1$ and $\hat{Y}_2$ are the estimated domain volumes in Ω_1 and Ω_2, respectively; and $\hat{Y}_3$ represents system heat flux.

V. Levenberg–Marquardt Method for Minimization

If the interfacial surfaces among three conductive bodies are governed by $2 \times J^2$ control points, Eq. (10) is minimized with respect to the estimated control points $B_{p,i,j}$ to obtain

$$\frac{\partial f[\Gamma_1(\mathbf{B}_1), \Gamma_2(\mathbf{B}_2)]}{\partial B_{p,i,j}} = \sum_{m=1}^{I} \left[\frac{\partial \hat{Y}_m}{\partial B_{p,i,j}} \right] [\hat{Y}_m - Y_m] = 0;$$
$$i = 1 \text{ to } J, \qquad j = 1 \text{ to } J, \qquad p = 1 \text{ or } 2 \tag{11}$$

Equation (11) is linearized by expanding $\hat{Y}_m(B_{1,i,j}, B_{2,i,j})$ in a Taylor series and retaining the first-order terms. A damping parameter λ^n is then added to the resulting expression to improve convergence, leading to the LMM [13] given by

$$(\mathbf{F} + \lambda^n \mathbf{I}) \Delta \mathbf{B}_p = \mathbf{D} \tag{12a}$$

where

$$\mathbf{F} = \mathbf{\Psi}^T \mathbf{\Psi} \tag{12b}$$

$$\mathbf{D} = \mathbf{\Psi}^T \Delta \tag{12c}$$

$$\Delta \mathbf{B}_p = \mathbf{B}_p^{n+1} - \mathbf{B}_p^n \tag{12d}$$

Here, $\mathbf{\Psi}$ denotes the Jacobian matrix and is defined as

$$\mathbf{\Psi} \equiv \partial \hat{\mathbf{Y}} / \partial \mathbf{B}_p^T \tag{13}$$

Equation (12a) is now written in a form suitable for iterative calculation as

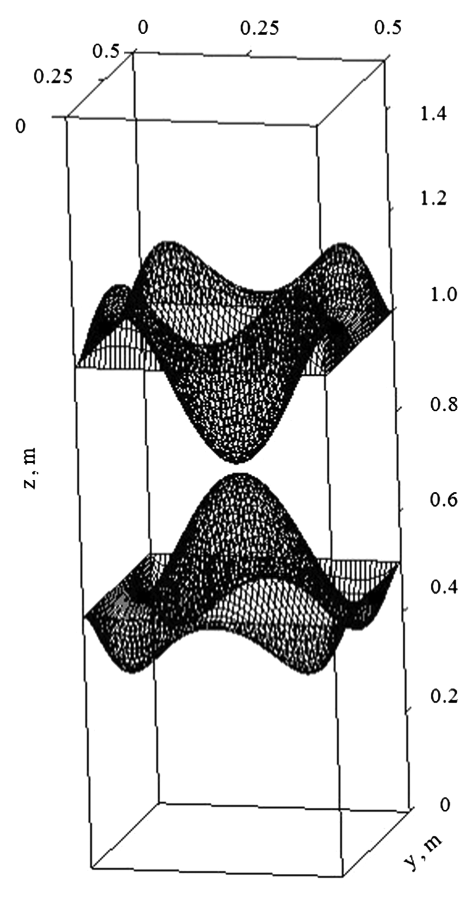

Fig. 1 Estimated interfacial surfaces with maximum (10.1%) increase in system heat flux in test case 1.

$$\mathbf{B}_p^{n+1} = \mathbf{B}_p^n + (\mathbf{\Psi}^T\mathbf{\Psi} + \lambda^n\mathbf{I})^{-1}\mathbf{\Psi}^T(\hat{Y}_m - Y_m) \tag{14}$$

The algorithm of choosing the damping value λ^n is previously described in detail, and the uniqueness of the inverse solutions by using LMM have been examined [13].

VI. Results and Discussion

The initial shape of interfacial surfaces $\Gamma_1(\mathbf{B}_1)$ and $\Gamma_2(\mathbf{B}_2)$ are assumed at the flat plates located at $z = 0.5$ and 1.0 m, respectively. In all the test cases considered here, we have chosen

$$L_1(\text{length in } x \text{ direction}) = L_2(\text{length in } y \text{ direction}) = 0.5 \text{ m}$$

$$L_3(\text{length in } z \text{ direction}) = 1.5 \text{ m}$$

$T_{\infty h} = 100°\text{C}$, $T_{\infty c} = 10°\text{C}$, and $h = 150 \text{ W/m}^2 \cdot \text{K}$. The grid numbers in the x, y, and z directions are taken to be $51 \times 51 \times 62$, respectively. The stopping criterion of the numerical test cases are defined as follows: when 1) the distance between two interfacial surfaces is less than 0.01 m or 2) the peak of interfacial surface between Ω_1 and Ω_2 to the domain boundary is less than 0.01 m, the calculations are terminated.

A. Numerical Test Case 1

In the existing heat transfer system, it is assumed that $k_1 = k_3 = 190 \text{ W/m} \cdot \text{K}$ (aluminum) and $k_2 = 48 \text{ W/m} \cdot \text{K}$ (steel), and the flat interfacial surfaces are located at $z = 0.5$ and 1.0 m, respectively. Based on the existing system, the system heat flux is calculated as $Q = 775$ W.

The optimal configurations of the interfacial surfaces are to be designed by varying the desired system heat fluxes, assuming that the boundary points of interfacial surfaces are always fixed at $z = 0.5$ and 1.0 m, respectively, and using 16 control points to describe each interfacial surface.

It is of interest to examine the design with a maximum increase in the system heat flux. When $Y_1 = Y_2 = 0.125 \text{ m}^3$ and $Y_3 = 930$ W are considered, the same volume constraint and a 20% increase in the total system heat flux result. After 13 iterations, the optimal shape of interfacial surfaces $\Gamma_1(\mathbf{B}_1)$ and $\Gamma_2(\mathbf{B}_2)$ can be obtained, and it is plotted in Fig. 1. In this ultimate case, $\hat{Y}_1$, $\hat{Y}_2$, and $\hat{Y}_3$ are calculated as 0.125 m^3, 0.125 m^3, and 853.3 W, respectively; only 10.1% of the total system heat flux can be increased in this case, and it defines the maximum possible system heat flux under the specified calculation conditions.

B. Numerical Test Case 2

It is of interest to find out what will happen if the magnitudes of the thermal conductivities are switched. The existing heat transfer system in test case 2 is assumed as $k_1 = k_3 = 48 \text{ W/m} \cdot \text{K}$ (aluminum) and $k_2 = 190 \text{ W/m} \cdot \text{K}$ (steel), and the flat interfacial surfaces are located at $z = 0.5$ and 1.0 m, respectively. Based on the preceding existing heat transfer system, the system heat flux is calculated as $Q = 611$ W.

When $Y_1 = Y_2 = 0.125 \text{ m}^3$ and $Y_3 = 794.3$ W are considered, the same volume constraint and a 30% increase in the total system heat flux result. After 17 iterations, the optimal shapes of interfacial surfaces $\Gamma_1(\mathbf{B}_1)$ and $\Gamma_2(\mathbf{B}_2)$ can be estimated, and they are plotted in Fig, 2. In this ultimate case, $\hat{Y}_1$, $\hat{Y}_2$, and $\hat{Y}_3$ are calculated as 0.125 m^3, 0.125 m^3, and 760.7 W, respectively; 24.5% of the total system heat flux can be increased in this case.

C. Numerical Test Case 3

In the third test case, the order of the thermal conductivity is rearranged as $k_1 = 48 \text{ W/m} \cdot \text{K}$ (steel), $k_2 = 76.8 \text{ W/m} \cdot \text{K}$ (iron),

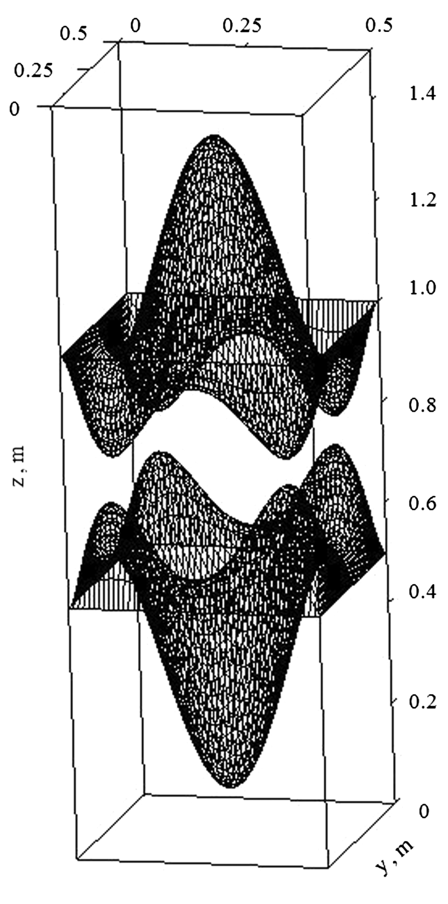

Fig. 2 Estimated interfacial surfaces with maximum (25.5%) increase in system heat flux in test case 2.

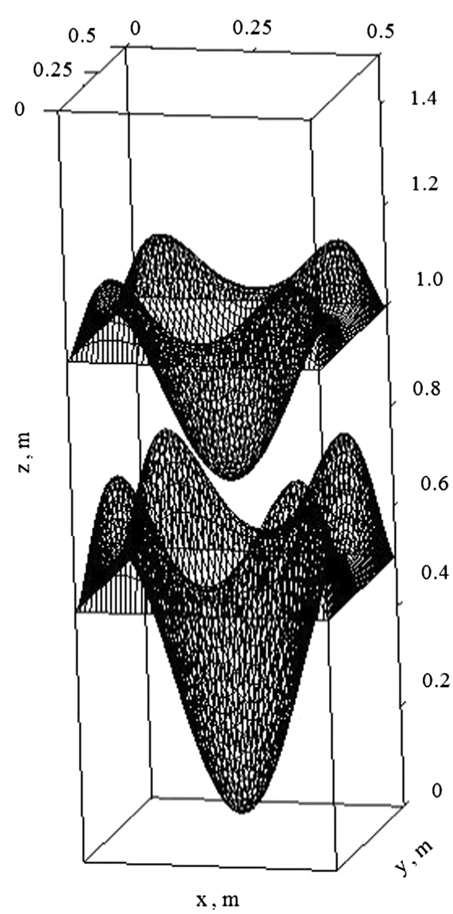

Fig. 3 Estimated interfacial surfaces with maximum (15.9%) increase in system heat flux in test case 3.

and $k_3 = 190\ \mathrm{W/m \cdot K}$ (aluminum). In the existing heat transfer system, the flat interfacial surfaces are located at $z = 0.5$ and 1.0 m, respectively. Based on the precedingexisting heat transfer system, the system heat flux is calculated as $Q = 682$ W.

When using $Y_1 = Y_2 = 0.125\ \mathrm{m}^3$ and $Y_3 = 818.4$ W, the same volume constraint and a 20% increase of the total system heat flux are considered. After 15 iterations, the optimal shapes of interfacial surfaces $\Gamma_1(\mathbf{B}_1)$ and $\Gamma_2(\mathbf{B}_2)$ are estimated and plotted in Fig. 3. In this ultimate case, $\hat{Y}_1$, $\hat{Y}_2$, and $\hat{Y}_3$ are calculated as $0.125\ \mathrm{m}^3$, $0.125\ \mathrm{m}^3$, and 790.4 W, respectively; that is, 15.9% of the total system heat flux can be increased in this case.

From the three numerical test cases, it can be concluded that the present algorithm was applied successfully to this three-dimensional shape design problem for determining two optimum interfacial surface configurations to obtain maximum system heat flux.

VII. Conclusions

The present three-dimensional shape design problem for estimating the shapes of the interfacial surfaces of a three-layer structure in a multiple region domain using the LMM was successfully examined using various numerical experiments. Three test cases involving different arrangements for thermal conductivities were considered. The results show that, by varying the shapes of interfacial surfaces, the system heat flux can indeed be increased and, for different combinations of thermal conductivities and the same volume constraint, the increases in the total system heat fluxes are between 10.1 and 24.5% for the cases considered in this study.

Acknowledgment

This work was supported in part through the National Science Council, grant number NSC-99-2221-E-006-238-MY3.

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